QUANTUM UNCERTAINT~~Y~~ ies

FRANCESCO BUSCEMI (NAGOYA UNIVERSITY)
COLLOQUIUM @ DEPT. APPLIED MATHEMATICS
HANYANG UNIVERSITY (ERICA)
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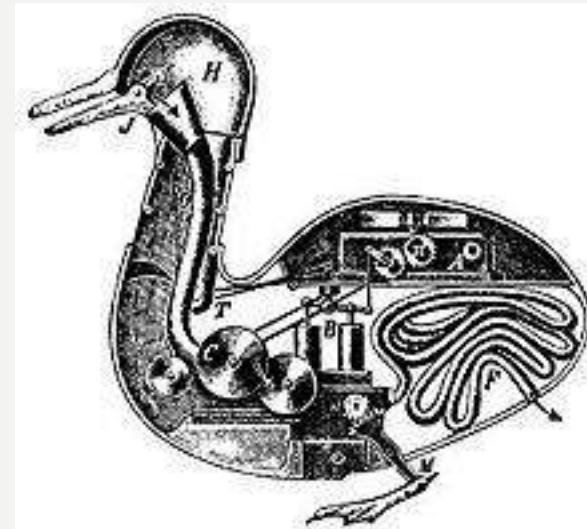
THE MECHANICAL CERTAINTY

"The universe as a clockwork"

An *orrery*.

The "digestive duck" (1739)

Laplace's Demon
"We may regard the present state of the universe as the effect of its past and the cause of its future.
An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future, just like the past, would be present before its eyes."

— Pierre Simon Laplace, *A Philosophical Essay on Probabilities* (1814)

THE QUANTUM UNCERTAINTIES

- LAPLACE'S DREAM IS IMPOSSIBLE NOT ONLY IN PRACTICE (COMPLEXITY, CHAOS, ETC), BUT ALSO *IN PRINCIPLE!*

- THE CULPRIT: INCOMPATIBILITY OF QUANTUM MEASUREMENTS

- WHAT ARE QUANTUM MEASUREMENTS? HOW TO DEFINE THEIR INCOMPATIBILITY? ARE THERE "MANY" QUANTUM UNCERTAINTIES?

THE INCEPTION OF QUANTUM THEORY

- 1900: Max Planck suggests that radiation is emitted in discrete packets, so called "quanta" (sing. quantum, from Latin)

- 1905: Einstein explains the photoelectric effects using quantized radiation

- 1913: Millikan shows that electric charge always comes in integer multiples of an elementary charge, the electron

- 1913: Bohr's atomic model: energy is also quantized

- 1923: De Broglie suggests the "wave-particle duality"

- 1926~1932: von Neumann lays the mathematical foundations of quantum theory

- 1927: Heisenberg's formulates the "uncertainty principle"
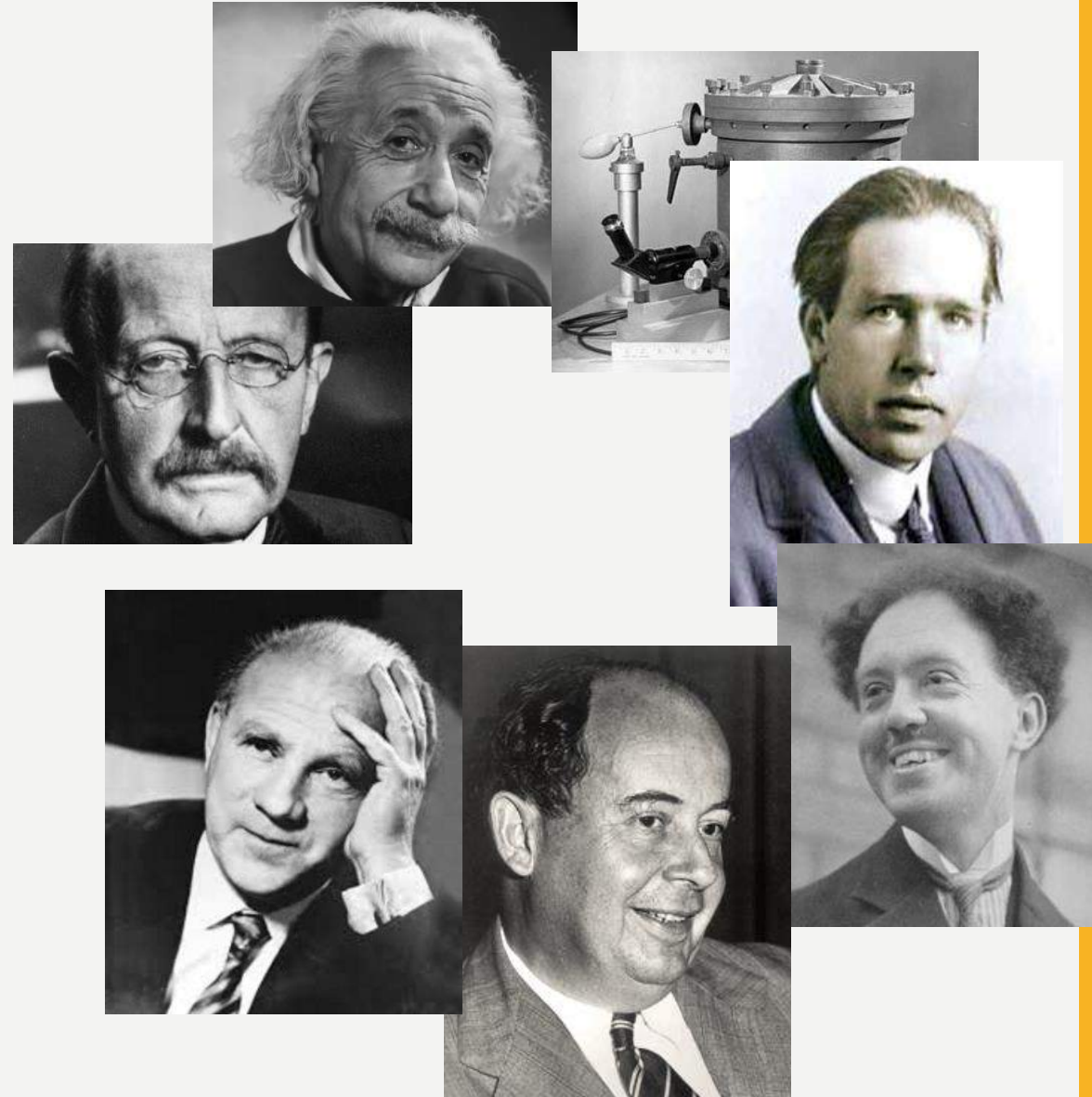
QUANTUM MEASUREMENTS

Quantum postulates:

- **Quantum states**:
  normalized complex vectors $|\psi\rangle$
  (here, Dirac notation: $(\phi, \psi) = \langle\phi|\psi\rangle$)

- **Schrödinger's equation**:
  a closed system's evolution is a unitary
  transformation

- **Quantum (projective) measurements**:
  given as set $\{m_i, \Pi_i\}_i$ where $m_i \in \mathbb{R}$ are the
  measurement's outcomes, and $\Pi_i \Pi_j = \delta_{ij} \Pi_i \geq 0$ are the measurement's operators

- **Born's rule**:
  on state $|\psi\rangle$, outcome m_i is obtained with
  probability $p_i = \|\Pi_i|\psi\rangle\|^2 = \langle\psi|\Pi_i|\psi\rangle$

- indeed, a measurement process is an open
  process!
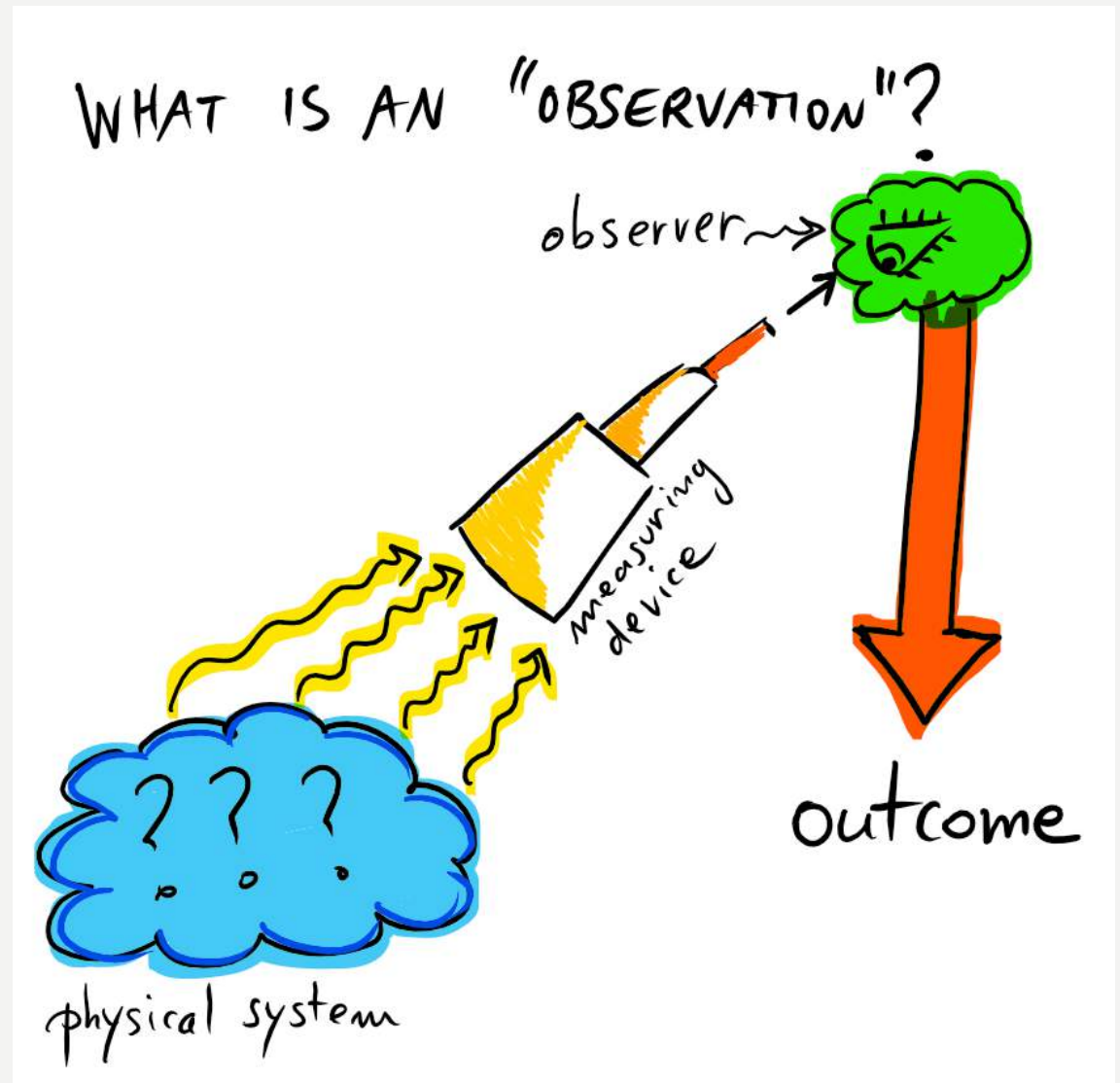
QUANTUM OBSERVABLES

Observables:

- Due to the "spectral theorem," projective measurements $\{m_i, \Pi_i\}_i$ are in one-to-one correspondence with self-adjoint operators $A = A^\dagger = \sum_i m_i \Pi_i$

- Self-adjoint operators are hence called "observables" because they correspond to "measurable quantities"

- Analogy with analytical mechanics: there, physical quantities were real functions; here they are self-adjoint operators ("functions of functions")
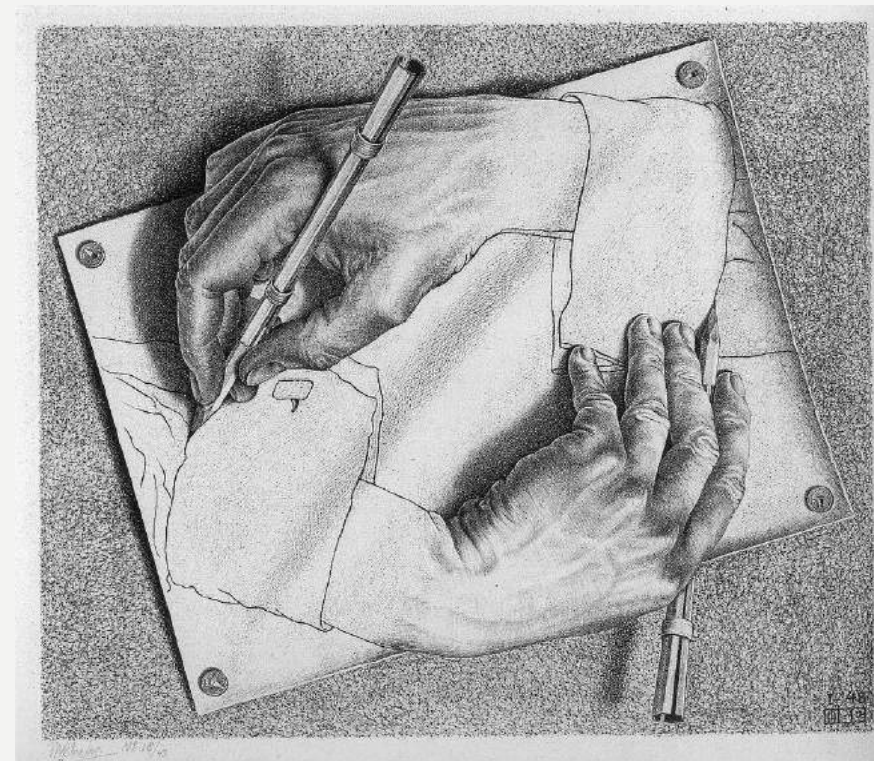

operation vs representation
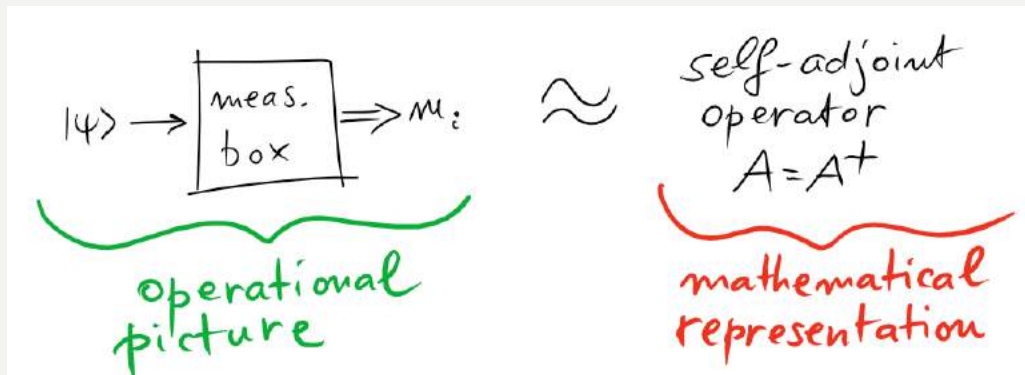
NONCOMMUTATIVITY

- Operators, in general, do non-commute: $AB - BA \neq 0$

- It is easy to understand the meaning of noncommutativity in the case of maps

- Quantum theory states that also physical quantities are represented by operators

- What does it mean that two physical quantities do not commute?

- In particular, what does noncommutativity mean in the context of the measurement process? Can we "see" noncommutativity?
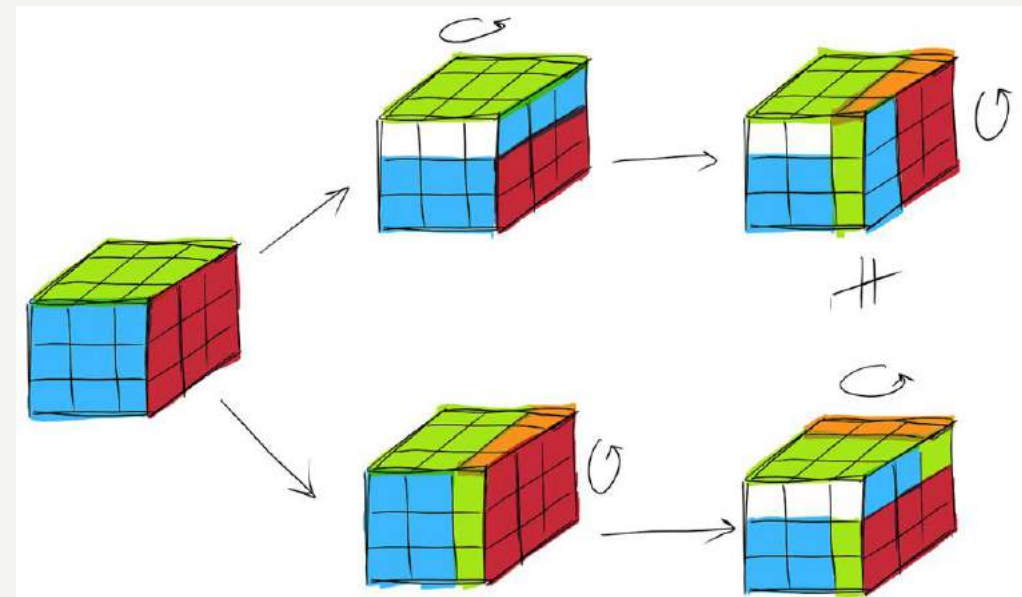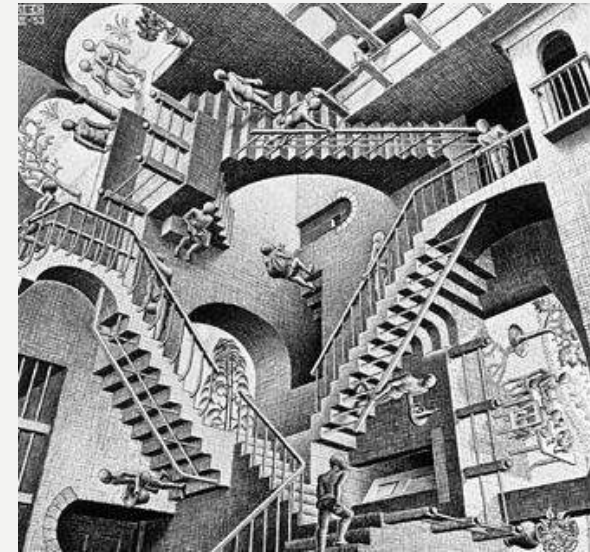
noncommutativity: example with rotations

HEISENBERG-ROBERTSON RELATIONS

- average value (expectation value):

$$\sum m_i p_i = \sum m_i \langle \psi | \Pi_i | \psi \rangle = \langle \psi | A | \psi \rangle = \langle A \rangle_\psi$$

- standard deviation (root variance):

$$\sigma_\psi(A) = \sqrt{\left\langle \psi \left| (A - \langle A \rangle_\psi)^2 \right| \psi \right\rangle} = \sqrt{\langle A^2 \rangle_\psi - \langle A \rangle_\psi^2}$$

- mathematical relation (Heisenberg-Robertson):

$$\sigma_\psi(A)\sigma_\psi(B) \geq \frac{1}{2} |\langle \psi | (AB - BA) | \psi \rangle|$$

- when $A = \hat{q}$ (position) and $B = \hat{p}$ (momentum), $\hat{q}\hat{p} - \hat{p}\hat{q} = i\hbar$, and

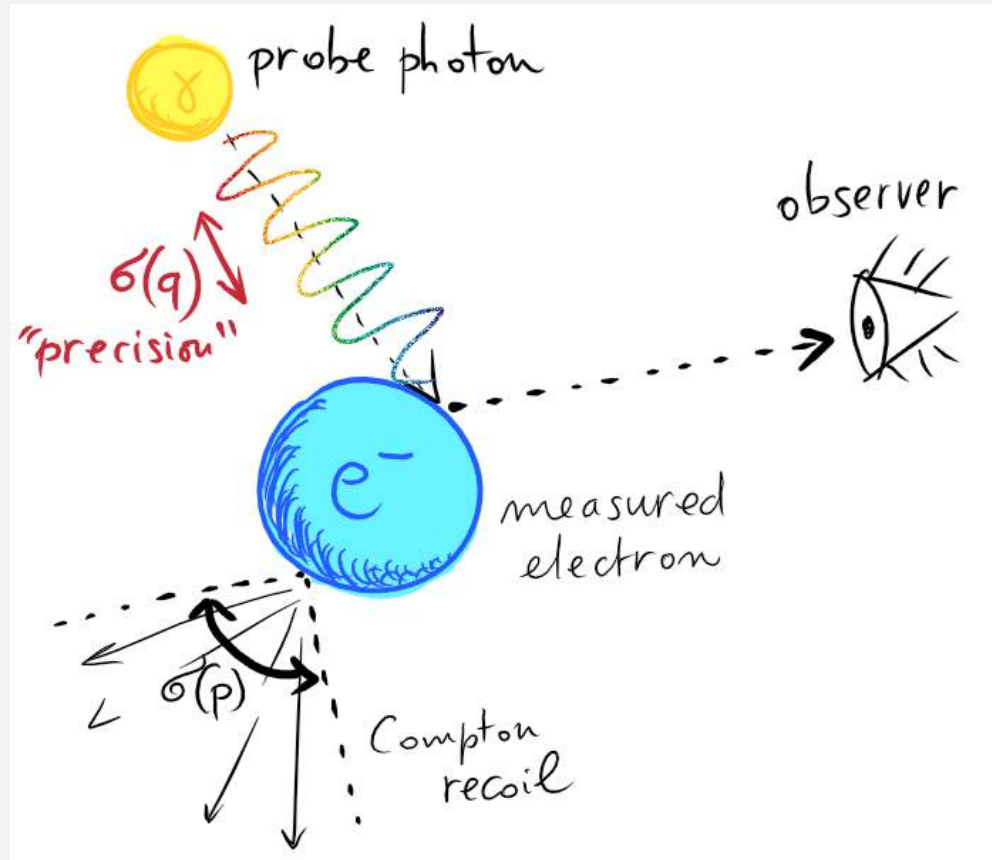$$\sigma_\psi(\hat{q})\sigma_\psi(\hat{p}) \geq \frac{\hbar}{2} \approx 10^{-34} J \cdot s$$

(notice the lower bound is here independent of ψ)

The question is: what are the "practical" consequences of this bound? Can we give an "intuitive" explanation?

HEISENBERG'S INTUITION

the gamma-ray "microscope"

$$\sigma(q)\sigma(p) \geq \frac{\hbar}{2} \approx 10^{-34} J \cdot s$$

Let $\sigma(q)$ be the precision with which the value q is known (i.e., the mean error of q), therefore here the wavelength of the light. Let $\sigma(p)$ be the precision with which the value p is determinable; that is, here, the discontinuous change of p in the Compton effect (scattering).

—W. Heisenberg, "The physical content of quantum kinematics and mechanics." 1927

Paraphrasing: "The more information we obtain about the electron's present position, the more uncertain the electron's future position becomes."

AGAINST FORMULATIONS "À LA HEISENBERG"

Criticisms to Heisenberg's argument (i.e., the interpretation):

- It is based on a semi-classical model for the measurement interaction (recoil)

- Heisenberg uses it to characterize the *measurement process*, but it actually refers to the *state preparation process*

- Orthodox interpretation: "no quantum state exists, which has both position and momentum sharply defined as the same time"

Criticisms to the Heisenberg-Robertson bound (i.e., the mathematics):

- Standard deviations depend too much on the precise numerical value of eigenvalues (scaling, relabeling)

- The bound in general depends on the state of the system: it becomes trivial if $|\psi\rangle$ is either eigenstate of A or of B (remember the important exception of position and momentum)

- Standard deviations do not really have an operational interpretation in information theory

SHANNON ENTROPY

"Information is the resolution of uncertainty."
-Claude Elwood Shannon
Father of Information Theory
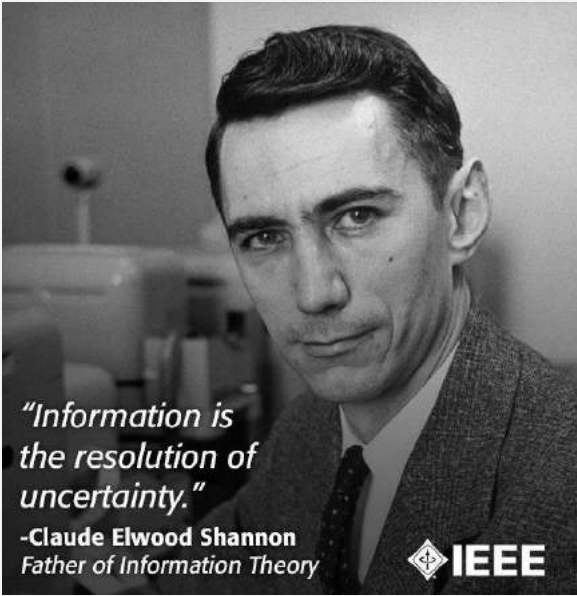
My greatest concern was what to call it. I thought of calling it "information," but the word was overly used, so I decided to call it "uncertainty." When I discussed it with John von Neumann, he had a better idea. Von Neumann told me, "You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, no one really knows what entropy really is, so in a debate you will always have the advantage."

- Given is a discrete random variable X, taking values x_i with probability $p(i)$
- Its entropy is given by $H(X) = -\sum_i p(i) \log_2 p(i)$
- Entropy measures how "random" the random variable is (compression rate, etc.)

Small variance "implies" small entropy, but not vice versa
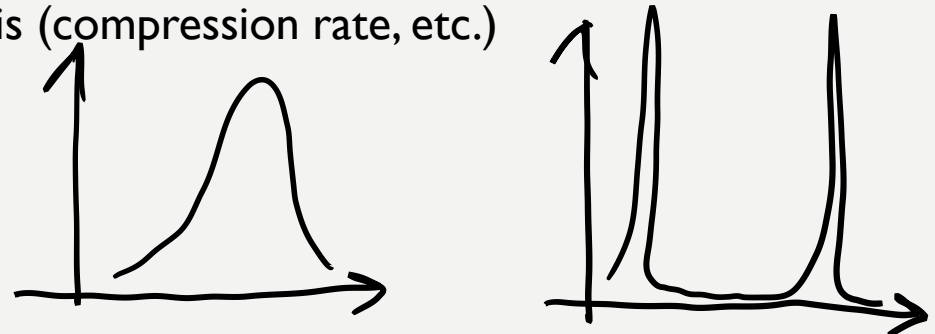
ENTROPIC UNCERTAINTY RELATIONS

- For simplicity, assume non-degenerate observables $A = \sum_i m_i |\alpha_i\rangle\langle\alpha_i|$ (i.e., $m_i \neq m_j$)

- Denote $H(A|\psi) = -\sum_i p(i) \log_2 p(i)$ where $p(i) = |\langle\alpha_i|\psi\rangle|^2$

- Then one has the following bound (Maassen, Uffink, 1988):

$$H(A|\psi) + H(B|\psi) \geq -\log_2 c(A, B)$$

where $c(A, B) = \max_{i,j} |\langle\alpha_i|\beta_j\rangle|^2$

- The lower bound does not depend on the state $|\psi\rangle$: automatically holds for *mixed* states too.
Compare this with what one would get from Robertson: $\sigma_\varrho(A)\sigma_\varrho(B) \geq \frac{1}{2}|Tr[\varrho(AB - BA)]|$

- Entropy has a neat operational meaning in information theory

WHAT ABOUT HEISENBERG'S ORIGINAL INTUITION?

- So, is it true or not, that information extraction causes disturbance?

- Difference between **static uncertainty principles** ("it is impossible to prepare a state that has all dynamical variables sharply distributed") and **dynamical uncertainty principles** ("the act of measuring one dynamical variable with high accuracy necessarily disturbs the others")

- Both the Heisenberg-Robertson relation and the Maassen-Uffink relation are static uncertainty relations

- **Dynamical uncertainty principles**: Ozawa (variance based, state dependent), Busch-Lahti-Werner (variance based, state independent), Buscemi-Hall-Ozawa-Wilde (entropic, state independent), Coles-Furrer (entropic, state dependent)

- …

CONCLUSIONS

- **Laplace (mechanics)**:
  "…for such an intellect nothing would be uncertain and the future, just like the past, would be present before its eyes."

- **Shannon (information theory)**:
  "…we may have knowledge of the past but cannot control it; we may control the future but have no knowledge of it."

- **Quantum theory (this talk)**:
  we cannot completely know the present (static uncertainty principles), and the more we learn about it, the more uncertain the future becomes (dynamical uncertainty principles)

- Is there any "deeper" description of reality then?