A resource theory of quantum nonlocality
(in space and time)

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Two paradigms for entanglement verification

Entanglement witnesses

\[ p(a, b) = \text{Tr} \left[ (P_A^a \otimes Q_B^b) \rho_{AB} \right] \]

- faithfulness: for any entangled state, there exists a witness detecting it

- measurement devices need to be perfect

Bell tests

\[ p(a, b|x, y) = \text{Tr} \left[ (P_{A|x}^a \otimes Q_{B|y}^b) \rho_{AB} \right] \]

- hidden nonlocality: some entangled states never violate any Bell inequality

- device independence
The time-like analogue: quantum memory verification

- the Choi correspondence, $\mathcal{E}_{A\rightarrow B} \leftrightarrow \rho_{AB}$, suggests trying the same approach in time.
- encouraging fact: "classical" (i.e., separable) states correspond to "classical" (i.e., entanglement-breaking) channels.

### Process tomography

$$p(b|x) = \text{Tr}[\mathcal{E}(\sigma_x) \ P_b]$$

### Time-like Bell tests

$$p(a, b|x, y) = \text{Tr}[\mathcal{E}(\sigma_{a|x}) \ P_{b|y,x,a}]$$

- in full analogy with entanglement witnesses, process tomography is faithful (😊) but requires complete trust in the tomographic devices (😭)
- instead, time-like Bell tests simply trivialize: $A$ can always signal to $B$. 
The case of *two* memories

✓ however, if *two* quantum memories are available, one can imagine doing the following

![Diagram of two quantum memories](image)

✓ here, we need two quantum memories, and the test is assessing *the pair simultaneously* (and it’s a Bell test, hence device-independent but not faithful)

✓ thus the problem remains: *is it possible to certify a single given memory, without using any side-channel?*
Let us go back to the space-like setting and try to modify Bell’s scenario...
The “semiquantum” Bell scenario

- in conventional nonlocal games, questions are classical labels; in semiquantum (nonlocal) games, questions are encoded on quantum states
- the referee chooses questions $x$ and $y$ at random
- the referee encodes questions on quantum states $\tau^x_A$, and $\omega^y_B$
- the system $A'$ is sent to Alice, $B'$ to Bob
- Alice and Bob must locally compute answers $a$ and $b$

Achievable correlations in the semiquantum scenario are given by

$$p(a, b|x, y, \rho_{AB}) = \text{Tr}[(P^a_{A'} \otimes Q^b_{B'B'}) (\tau^x_A \otimes \rho_{AB} \otimes \omega^y_B)]$$

for varying POVMs
Semiquantum nonlocal games

- in analogy with quantum statistical decision problems (Holevo, 1973), we also introduce a real-valued payoff function $f(a, b, x, y)$
- the “utility” of a given bipartite state $\rho_{AB}$ w.r.t. the semiquantum nonlocal game $(\tau^x, \omega^y, f)$ is then computed as

$$f^*(\rho_{AB}) = \max_{P, Q} \sum_{a, b, x, y} f(a, b, x, y) \text{Tr} \left[ (P^a_{A'} \otimes Q^b_{BB'}) (\tau^{x}_{A'} \otimes \rho_{AB} \otimes \omega^{y}_{B'}) \right] p(a, b | x, y, \rho_{AB})$$

Theorem (2012)

*Given two bipartite states $\rho_{AB}$ and $\sigma_{CD}$, $f^*(\rho_{AB}) \geq f^*(\sigma_{CD})$ for all semiquantum nonlocal games, if and only if*

$$\sigma_{CD} = \sum_{\lambda} p(\lambda) \left[ \mathcal{E}_A^{\lambda} \otimes \mathcal{F}_{B \rightarrow D}^{\lambda} \right] (\rho_{AB}) ,$$

*for some CPTP maps $\mathcal{E}, \mathcal{F}$ and normalized probability distribution $p(\lambda)$. *
A resource theory of *quantum nonlocality*

- semiquantum nonlocal games provide a complete set of monotones for *local operations and shared randomness* (LOSR)

- it is natural to understand this as a resource theory of *quantum nonlocality*: free operations are LOSR and hence free states are separable states

- this is different from a resource theory of nonlocality (without “quantum”): there, being manipulated are correlations $p(a, b|x, y)$ (like, e.g., PR-boxes), not bipartite quantum states $\rho_{AB}$
Robustness properties of semiquantum nonlocal games

- semiquantum nonlocal games \(\rightsquigarrow\) measurement-device-independent entanglement witnesses
- in particular, robust against losses in the detectors (losses spoil Bell tests)
- moreover, robust against classical communication between players (this also spoils Bell tests)
- this feature is especially welcome in the time-like scenario, where signaling cannot be ruled out and hence must be assumed

While we do not have time-like Bell tests, we could have time-like semiquantum tests!
It! Could! Work!
The time-like semiquantum scenario

- give Alice a state $\tau^x$ at time $t_0$
- wait some time
- give her another state $\omega^y$ at time $t_1$
- the round ends with Alice outputting an outcome $b$

(here we should think of $B$ as “Alice after some time”)

Achievable input/output correlations are computed as

$$p(b|x, y, \mathcal{N}) = \sum_i \text{Tr} \left[ P^b_{BB} \{ \omega^y_B \otimes (\mathcal{N}_{A\rightarrow B} \circ \mathcal{I}^i_{A\rightarrow A}) (\tau^x_A) \} \right]$$

where $\{\mathcal{I}^i\}$ is an instrument, so that any amount of classical communication can be transmitted via the index $i$. 

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Time-like semiquantum games

- introduce a real-valued payoff function \( f(b, x, y) \)
- the utility of a channel \( \mathcal{N} \) is given by

\[
f^*(\mathcal{N}) = \max_{\mathcal{I},\mathcal{P}} \sum_{b,x,y} f(b, x, y) \sum_i \text{Tr} \left[ P_{BB}^{b|i} \left\{ \omega_B^y \otimes (\mathcal{N}_{A \rightarrow B} \circ \mathcal{I}_{A \rightarrow A}^i) (\tau_A^x) \right\} \right] \quad \sum_i \sum_{b,x,y} f(b, x, y) \sum_i \text{Tr} \left[ P_{BB}^{b|i} \left\{ \omega_B^y \otimes (\mathcal{N}_{A \rightarrow B} \circ \mathcal{I}_{A \rightarrow A}^i) (\tau_A^x) \right\} \right]
\]

Theorem (2018)

Given two channels \( \mathcal{N}_{A \rightarrow B} \) and \( \mathcal{N}'_{A' \rightarrow B'} \), \( f^*(\mathcal{N}) \geq f^*(\mathcal{N}') \) for all time-like semiquantum games, if and only if

\[
\mathcal{N}'_{A' \rightarrow B'} = \sum_i \mathcal{D}_{B \rightarrow B'}^i \circ \mathcal{N}_{A \rightarrow B} \circ \mathcal{I}_{A' \rightarrow A}^i,
\]

for some instrument \( \{\mathcal{I}^i\} \) and CPTP maps \( \{\mathcal{D}^i\} \).
A resource theory of quantum memories

- free operations are given by classically correlated pre/post-processing maps (i.e., quantum combs with classical memory)
- free “states” are entanglement-breaking channels
- no shared entanglement or backward classical communication in the case of memories
as long as the quantum memory (channel) $\mathcal{E}$ is not entanglement breaking, there exists a time-like semiquantum game capable of certifying that

assumption: we need to trust the preparation of states $\tau^x$ and $\omega^y$, but that is anyway required in the time-like scenario (no fully device-independent quantum channel verification [Pusey, 2015])

faithfulness with minimal assumptions

extra feature: it is possible to quantify the minimal dimension of the quantum memory
Conclusions

- entanglement witnesses: faithful, but complete trust is necessary
- Bell tests: fully device-independent, but not faithful
- semiquantum tests: faithful, and trust is required only for the referee’s preparation devices
- semiquantum tests are particularly compelling in the time-like scenario, in which no device-independent quantum channel verification exists anyway
- verification of non-classical correlations among any two locally quantum agents, independent of their causal separation
- the test is quantitative: a lower bound on the quantum dimension can be given