The Theory of Statistical Comparison in Quantum Information and Foundations

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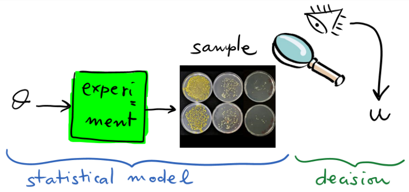
The Birth in Mathematical Statistics

Statistical Decision Problems



$$\begin{tikzcd}
\Theta \arrow[r, "\text{experiment}"] \arrow[d, squiggly] & \mathcal{X} \arrow[r, "\text{decision}"] \arrow[d, squiggly] & \mathcal{U} \arrow[d, squiggly] \\
\theta \arrow[r, "w(x|\theta)"'] & x \arrow[r, "d(u|x)"'] & u
\end{tikzcd}$$

payoff is $\ell(\theta, u) \in \mathbb{R}$

Definition (Statistical Models and Decisions Problems)

A statistical experiment (i.e., statistical model) is a triple $\langle \Theta, \mathcal{X}, w \rangle$, a statistical decision problem (i.e., statistical game) is a triple $\langle \Theta, \mathcal{U}, \ell \rangle$.

How Much Is an Experiment Worth?

- the experiment *is given*, i.e., it is the “resource”
- the decision instead *can be optimized*

$$\begin{array}{ccccc}
\Theta & \xrightarrow{\text{experiment}} & \mathcal{X} & \xrightarrow{\text{decision}} & \mathcal{U} \\
\rightsquigarrow & & \rightsquigarrow & & \rightsquigarrow \\
\theta & \xrightarrow[w(x|\theta)]{} & x & \xrightarrow[d(u|x)]{} & u
\end{array}$$

Definition (Expected Payoff)

The expected payoff of a statistical experiment $\mathbf{w} = \langle \Theta, \mathcal{X}, w \rangle$ w.r.t. a decision problem $\langle \Theta, \mathcal{U}, \ell \rangle$ is given by

$$\mathbb{E}_{\langle \Theta, \mathcal{U}, \ell \rangle}[\mathbf{w}] \stackrel{\text{def}}{=} \max_{d(u|x)} \sum_{u,x,\theta} \ell(\theta, u) d(u|x) w(x|\theta) |\Theta|^{-1} .$$

Comparing Experiments 1/2

First experiment:
$\mathbf{w} = \langle \Theta, \mathcal{X}, w(x|\theta) \rangle$

$$\begin{tikzcd} \Theta \arrow[r, "\text{experiment}"] \arrow[d, squiggly] & \mathcal{X} \arrow[r, "\text{decision}"] \arrow[d, squiggly] & \mathcal{U} \arrow[d, squiggly] \\ \theta \arrow[r, "w(x|\theta)"'] & x \arrow[r, "d(u|x)"'] & u \end{tikzcd}$$

Second experiment:
$\mathbf{w}' = \langle \Theta, \mathcal{Y}, w'(y|\theta) \rangle$

$$\begin{tikzcd} \Theta \arrow[r, "\text{experiment}"] \arrow[d, squiggly] & \mathcal{Y} \arrow[r, "\text{decision}"] \arrow[d, squiggly] & \mathcal{U} \arrow[d, squiggly] \\ \theta \arrow[r, "w'(y|\theta)"'] & x \arrow[r, "d'(u|y)"'] & u \end{tikzcd}$$

If $\mathbb{E}_{\langle \Theta, \mathcal{U}, \ell \rangle}[\mathbf{w}] \geq \mathbb{E}_{\langle \Theta, \mathcal{U}, \ell \rangle}[\mathbf{w}']$, then experiment $\langle \Theta, \mathcal{X}, w \rangle$ is better than experiment $\langle \Theta, \mathcal{Y}, w' \rangle$ for problem $\langle \Theta, \mathcal{U}, \ell \rangle$.

Comparing Experiments 2/2

Definition (Information Preorder)

If the experiment $\langle \Theta, \mathcal{X}, w \rangle$ is better than experiment $\langle \Theta, \mathcal{Y}, w' \rangle$ **for all** decision problems $\langle \Theta, \mathcal{U}, \ell \rangle$, then we say that $\langle \Theta, \mathcal{X}, w \rangle$ is *more informative* than $\langle \Theta, \mathcal{Y}, w' \rangle$, and write

$$\langle \Theta, \mathcal{X}, w \rangle \succeq \langle \Theta, \mathcal{Y}, w' \rangle \, .$$

Problem. The information preorder is operational, but not really "concrete". Can we visualize this better?

Blackwell's Theorem (1948-1953)

Blackwell-Sherman-Stein Theorem

Given two experiments with the same parameter space, $\langle \Theta, \mathcal{X}, w \rangle$ and $\langle \Theta, \mathcal{Y}, w' \rangle$, the condition $\langle \Theta, \mathcal{X}, w \rangle \succeq \langle \Theta, \mathcal{Y}, w' \rangle$ holds *iff* there exists a conditional probability $\varphi(y|x)$ such that $w'(y|\theta) = \sum_x \varphi(y|x) w(x|\theta)$.

$$
\begin{tikzcd}
\Theta \arrow[r] \arrow[d, rightsquigarrow] & \mathcal{Y} \arrow[d, rightsquigarrow] \\
\theta \arrow[r, "w'(y|\theta)"'] & y
\end{tikzcd}
\quad = \quad
\begin{tikzcd}
\Theta \arrow[r] \arrow[d, rightsquigarrow] & \mathcal{X} \arrow[r, "\text{noise}"] \arrow[d, rightsquigarrow] & \mathcal{Y} \arrow[d, rightsquigarrow] \\
\theta \arrow[r, "w(x|\theta)"'] & x \arrow[r, "\varphi(y|x)"'] & y
\end{tikzcd}
$$

David H. Blackwell (1919-2010)

The Precursor: Majorization

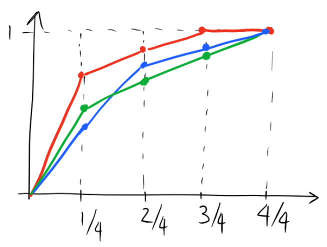
Lorenz Curves and Majorization

- two probability distributions, $\boldsymbol{p}$ and $\boldsymbol{q}$, of the same dimension n
- truncated sums $P(k) = \sum_{i=1}^{k} p_i^{\downarrow}$ and $Q(k) = \sum_{i=1}^{k} q_i^{\downarrow}$, for all $k = 1, \ldots, n$
- $\boldsymbol{p}$ majorizes $\boldsymbol{q}$, i.e., $\boldsymbol{p} \succeq \boldsymbol{q}$, whenever $P(k) \geq Q(k)$, for all k
- minimal element: uniform distribution $\boldsymbol{e} = n^{-1}(1, 1, \cdots, 1)$

Hardy, Littlewood, and Pólya

$\boldsymbol{p} \succeq \boldsymbol{q} \iff \boldsymbol{q} = M\boldsymbol{p}$, for some bistochastic matrix M.

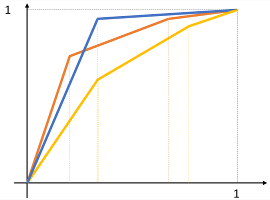
Lorenz curve for probability distribution $\boldsymbol{p} = (p_1, \cdots, p_n)$:



$$(x_k, y_k) = (k/n, P(k)), \quad 1 \leq k \leq n$$

Generalization: Relative Lorenz Curves

- two *pairs* of probability distributions, $(\boldsymbol{p}_1, \boldsymbol{p}_2)$ and $(\boldsymbol{q}_1, \boldsymbol{q}_2)$, of dimension m and n, respectively
- relabel entries such that ratios p_1^i / p_2^i and q_1^j / q_2^j are nonincreasing
- construct the truncated sums $P_{1,2}(k) = \sum_{i=1}^{k} p_{1,2}^i$ and $Q_{1,2}(k)$
- $(\boldsymbol{p}_1, \boldsymbol{p}_2) \succeq (\boldsymbol{q}_1, \boldsymbol{q}_2)$ iff the relative Lorenz curve of the former is never below that of the latter



Relative Lorenz curves:

$(x_k, y_k) = (P_2(k), P_1(k))$

Blackwell (Theorem for Dichotomies)

$(\boldsymbol{p}_1, \boldsymbol{p}_2) \succeq (\boldsymbol{q}_1, \boldsymbol{q}_2) \iff \boldsymbol{q}_i = M \boldsymbol{p}_i$, for some stochastic matrix M.

Extension to the Quantum Case

Quantum Decision Theory

A.S. Holevo, *Statistical Decision Theory for Quantum Systems*, 1973.

classical case	**quantum case**
• decision problems $\langle \Theta, \mathcal{U}, \ell \rangle$	• decision problems $\langle \Theta, \mathcal{U}, \ell \rangle$
• experiments $\mathbf{w} = \langle \Theta, \mathcal{X}, \{w(x\|\theta)\} \rangle$	• quantum experiments $\mathcal{E} = \left\langle \Theta, \mathcal{H}_S, \{\rho_S^\theta\} \right\rangle$
• decisions $d(u\|x)$	• POVMs $\{P_S^u : u \in \mathcal{U}\}$
• $p_c(u,\theta) = \sum_x d(u\|x) w(x\|\theta) \|\Theta\|^{-1}$	• $p_q(u,\theta) = \mathrm{Tr}\left[\rho_S^\theta \ P_S^u\right] \|\Theta\|^{-1}$
• $\mathbb{E}_{\langle \Theta, \mathcal{U}, \ell \rangle}[\mathbf{w}] = \max_{d(u\|x)} \sum \ell(\theta,u) p_c(u,\theta)$	• $\mathbb{E}_{\langle \Theta, \mathcal{U}, \ell \rangle}[\mathcal{E}] = \max_{\{P_S^u\}} \sum \ell(\theta,u) p_q(u,\theta)$

Hence, it is possible, for example, to compare quantum experiments with classical experiments, and introduce the information preorder as done before.

Example: Semiquantum Blackwell Theorem

Theorem (FB, 2012)

Consider two quantum experiments $\mathcal{E} = \left\langle \Theta, \mathcal{H}_S, \{\rho_S^\theta\} \right\rangle$ and $\mathcal{E}' = \left\langle \Theta, \mathcal{H}_{S'}, \{\sigma_{S'}^\theta\} \right\rangle$, and assume that the σ's all commute. Then, $\mathcal{E} \succeq \mathcal{E}'$ holds *iff* there exists a quantum channel (CPTP map) $\Phi : \mathcal{L}(\mathcal{H}_S) \to \mathcal{L}(\mathcal{H}_{S'})$ such that $\Phi(\rho_S^\theta) = \sigma_{S'}^\theta$, for all $\theta \in \Theta$.

Developments

- **fully quantum** information preorder
- quantum relative majorization
- statistical comparison of **quantum measurements** (compatibility preorder)
- statistical comparison of **quantum channels** (input-degradability preorder, output-degradability preorder, coding preorder, etc)
- applications: **quantum information theory**, quantum thermodynamics, open quantum systems dynamics, quantum resource theories, **quantum foundations**, ...

The Viewpoint of Communication Theory

Statistics vs Information Theory

Statistical theory: Nature does not bother with coding

$$
\begin{array}{ccccc}
\Theta & \xrightarrow{\text{experiment}} & \mathcal{X} & \xrightarrow{\text{decision}} & \mathcal{U} \\
\rightsquigarrow & & \rightsquigarrow & & \rightsquigarrow \\
\theta & \xrightarrow[w(x|\theta)]{} & x & \xrightarrow[d(u|x)]{} & u
\end{array}
$$

Communication theory: a sender, instead, *does code*

$$
\begin{array}{ccccccc}
\mathcal{M} & \xrightarrow{\text{encoding}} & \Theta & \xrightarrow{\text{channel}} & \mathcal{X} & \xrightarrow{\text{decoding}} & \mathcal{U} \\
\rightsquigarrow & & \rightsquigarrow & & \rightsquigarrow & & \rightsquigarrow \\
m & \xrightarrow[e(\theta|m)]{} & \theta & \xrightarrow[w(x|\theta)]{} & x & \xrightarrow[d(u|x)]{} & u
\end{array}
$$

From Decision Problems to Decoding Problems

Definition (Decoding Problems)

Given a channel $\langle \mathcal{X}, \mathcal{Y}, w(y|x) \rangle$, a decoding problem is defined by an encoding $\langle \mathcal{M}, \mathcal{X}, e(x|m) \rangle$ and the payoff function is the optimum guessing probability:

$$\mathbb{E}_{\langle \mathcal{M}, \mathcal{X}, e(x|m) \rangle}[\langle \mathcal{X}, \mathcal{Y}, w(y|x) \rangle] \stackrel{\text{def}}{=} \max_{d(m|y)} \sum_{m,x,y} d(m|y) w(y|x) e(x|m) |\mathcal{M}|^{-1}$$
$$= 2^{-H_{\min}(M|Y)}$$

$$\begin{tikzcd}
\mathcal{M} \arrow[r, "\text{encoding}"] \arrow[d, squiggly] & \mathcal{X} \arrow[r, "\text{channel}"] \arrow[d, squiggly] & \mathcal{Y} \arrow[r, "\text{decoding}"] \arrow[d, squiggly] & \mathcal{M} \arrow[d, squiggly] \\
m \arrow[r, "e(x|m)"'] & x \arrow[r, "w(y|x)"'] & y \arrow[r, "d(\hat{m}|y)"'] & \hat{m}
\end{tikzcd}$$

Comparison of Classical Noisy Channels

$$\begin{tikzcd} \mathcal{M} \arrow[r, "\text{encoding}"] \arrow[d, rightsquigarrow] & \mathcal{X} \arrow[r, "\text{channel}"] \arrow[d, rightsquigarrow] & \mathcal{Y} \arrow[d, rightsquigarrow] \\ m \arrow[r, "e(x|m)"'] & x \arrow[r, "w(y|x)"'] & y \end{tikzcd}$$

$$\begin{tikzcd} \mathcal{M} \arrow[r, "\text{encoding}"] \arrow[d, rightsquigarrow] & \mathcal{X} \arrow[r, "\text{channel}"] \arrow[d, rightsquigarrow] & \mathcal{Z} \arrow[d, rightsquigarrow] \\ m \arrow[r, "e(x|m)"'] & x \arrow[r, "w'(z|x)"'] & z \end{tikzcd}$$

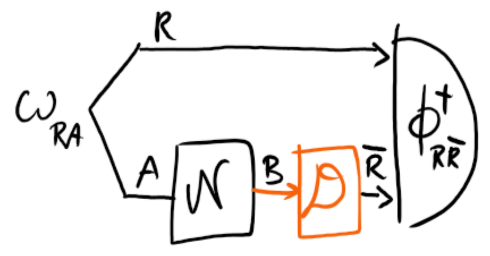
Theorem (FB, 2016)

The following are equivalent:

1. *there exists $\varphi(z|y)$: $w'(x|z) = \sum_y \varphi(z|y)w(y|x)$ (stochastic degradability);*
2. *for all codes $\langle \mathcal{M}, \mathcal{X}, e(x|m) \rangle$, $H_{\min}(M|Y) \leq H_{\min}(M|Z)$ (ambiguity preorder).*

The above strictly imply $H(M|Y) \leq H(M|Z)$ (Körner's and Marton's noisiness preorder).

Decoding Quantum Codes



Definition (Quantum Decoding Problems)

Given a quantum channel $\mathcal{N} : A \to B$, a quantum decoding problem is defined by a bipartite state ω_{RA} and the payoff function is the optimum singlet fraction:

$$\mathbb{E}_\omega[\mathcal{N}] \stackrel{\text{def}}{=} \max_{\mathcal{D}} \langle \Phi^+_{R\bar{R}} | (\mathrm{id}_R \otimes \mathcal{D}_{B \to \bar{R}} \circ \mathcal{N}_{A \to B})(\omega_{RA}) | \Phi^+_{R\bar{R}} \rangle$$

Comparison of Quantum Noisy Channels

Theorem

Given two quantum channels $\mathcal{N} : A \to B$ and $\mathcal{N}' : A \to B'$, the following are equivalent:

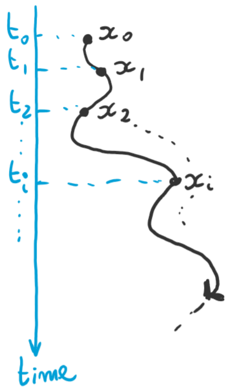
1. *there exists CPTP map $\mathcal{C}$: $\mathcal{N}' = \mathcal{C} \circ \mathcal{N}$ (degradability preorder);*
2. *for any bipartite state ω_{RA}, $\mathbb{E}_\omega[\mathcal{N}] \geq \mathbb{E}_\omega[\mathcal{N}']$ (coherence preorder);*
3. *for any bipartite state ω_{RA},* $H_{\min}(R|B)_{(\mathsf{id}\otimes\mathcal{N})(\omega)} \leq H_{\min}(R|B')_{(\mathsf{id}\otimes\mathcal{N}')(\omega)}$.

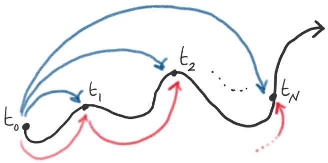
$\rightsquigarrow$ by adding symmetry constraints, we have applications in quantum thermodynamics

Application to Open Quantum Systems Dynamics

Discrete-Time Stochastic Processes

- Let x_i, for $i = 0, 1, \ldots$, index the state of a system at time $t = t_i$
- if the system can be initialized at time $t = t_0$, the process is fully described by the conditional distribution $p(x_N, \ldots, x_1 | x_0)$
- if the system evolving is quantum, we only have a quantum dynamical mapping $\left\{\mathcal{N}^{(i)}_{Q_0 \to Q_i}\right\}_{i=1,\ldots,N}$
- the process is divisible if there exist channels $\mathcal{D}^{(i)}$ such that $\mathcal{N}^{(i+1)} = \mathcal{D}^{(i)} \circ \mathcal{N}^{(i)}$ for all i



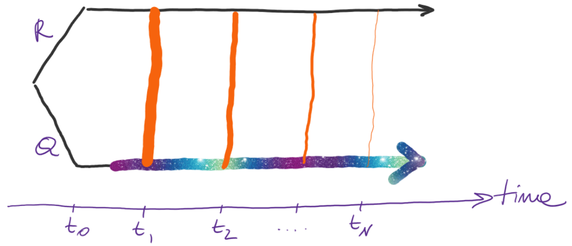


Divisibility as “Entanglement Flow”

Theorem (2016-2018)

Given an initial open quantum system Q_0, a quantum dynamical mapping $\left\{\mathcal{N}^{(i)}_{Q_0\to Q_i}\right\}_{i\geq 1}$ is divisibile if and only if, for any initial state ω_{RQ_0},

$$H_{\min}(R|Q_1) \leq H_{\min}(R|Q_2) \leq \cdots \leq H_{\min}(R|Q_N) \ .$$



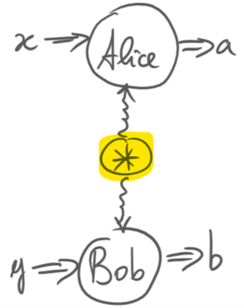
Application to Quantum Foundations: Probing Quantum Correlations in Space-Time

Part One: Quantum Space-Like Correlations

- nonlocal games (Bell tests) can be seen as bipartite decision problems $\langle \mathcal{X}, \mathcal{Y}; \mathcal{A}, \mathcal{B}; \ell \rangle$ played "in parallel" by non-communicating players

- with a classical source, $p_c(a, b|x, y) = \sum_\lambda \pi(\lambda) d_A(a|x, \lambda) d_B(b|y, \lambda)$

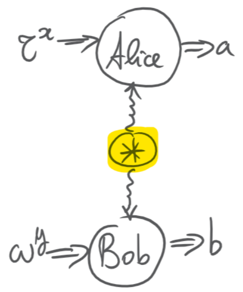
- with a quantum source, $p_q(a, b|x, y) = \mathrm{Tr}\left[\rho_{AB}\ (P_A^{a|x} \otimes Q_B^{b|y})\right]$



$$\mathbb{E}_{\langle \mathcal{X}, \mathcal{Y}; \mathcal{A}, \mathcal{B}; \ell \rangle}[*] \stackrel{\text{def}}{=} \max \sum_{x,y,a,b} \ell(x, y; a, b) p_{c/q}(a, b|x, y) |\mathcal{X}|^{-1} |\mathcal{Y}|^{-1}$$

Semiquantum Nonlocal Games

- **semiquantum nonlocal games** replace classical inputs with quantum inputs: $\langle \{\tau^x\}, \{\omega^y\}; \mathcal{A}, \mathcal{B}; \ell \rangle$
- with a classical source, $p_c(a,b|x,y) = \sum_\lambda \pi(\lambda) \operatorname{Tr}\left[(\tau_X^x \otimes \omega_Y^y)\ (P_X^{a|\lambda} \otimes Q_Y^{b|\lambda}) \right]$
- with a quantum source, $p_q(a,b|x,y) = \operatorname{Tr}\left[(\tau_X^x \otimes \rho_{AB} \otimes \omega_Y^y)\ (P_{XA}^a \otimes Q_{BY}^b) \right]$



$$\mathbb{E}_{\langle \{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle}[*] \stackrel{\text{def}}{=} \max \sum_{x,y,a,b} \ell(x,y;a,b) p_{c/q}(a,b|x,y) |\mathcal{X}|^{-1} |\mathcal{Y}|^{-1}$$

A Blackwell Theorem for Bipartite States

Theorem (FB, 2012)

Given two bipartite states ρ_{AB} and $\sigma_{A'B'}$, the condition (i.e., "nonlocality preorder")

$$\mathbb{E}_{\langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle}[\rho_{AB}] \geq \mathbb{E}_{\langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle}[\sigma_{A'B'}]$$

holds for all semiquantum nonlocal games, iff there exist CPTP maps $\Phi^{\lambda}_{A\to A'}$, $\Psi^{\lambda}_{B\to B'}$, and distribution $\pi(\lambda)$ such that

$$\sigma_{A'B'} = \sum_{\lambda} \pi(\lambda)(\Phi^{\lambda}_{A\to A'} \otimes \Psi^{\lambda}_{B\to B'})(\rho_{AB}) \ .$$

Corollaries

- For any separable state ρ_{AB},

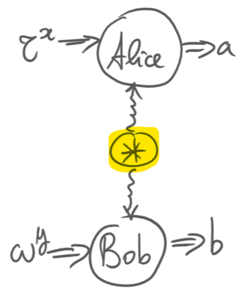
$$\mathbb{E}_{\langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle}[\rho_{AB}] = \mathbb{E}_{\langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle}[\rho_A \otimes \rho_B] = \mathbb{E}^{\mathsf{sep}}_{\langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle} ,$$

  for all semiquantum nonlocal games.
- For any entangled state ρ_{AB}, **there exists** a semiquantum nonlocal game $\langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle$ such that

$$\mathbb{E}_{\langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle}[\rho_{AB}] > \mathbb{E}^{\mathsf{sep}}_{\langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle} .$$

Other Properties of Semiquantum Nonlocal Games

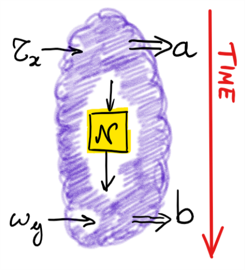
- can be considered as measurement device-independent entanglement witnesses (i.e., MDI-EW)
- can withstand losses in the detectors
- can withstand any amount of classical communication exchanged between Alice and Bob (not so conventional nonlocal games!)



Part Two:
Quantum Time-Like Correlations

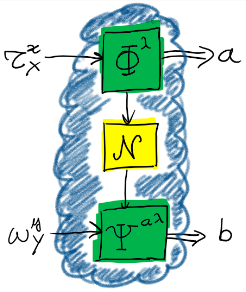
Semiquantum Nonlocality "in Time"

- Alice–Bob becomes 'Alice now'–'Alice later'
- with unlimited classical memory, $p_c(a, b|x, y) = \sum_\lambda \pi(\lambda) \operatorname{Tr}\left[\tau_X^x \ P_X^{a|\lambda}\right] \operatorname{Tr}\left[\omega_Y^y \ Q_Y^{b|a,\lambda})\right]$
- if, moreover, a quantum memory $\mathcal{N} : A \to B$ is available?



Admissible Quantum Strategies

- τ_X^x is fed through an *instrument* $\{\Phi_{X\to A}^{a|\lambda}\}$, and outcome a is recorded
- the quantum output of the instrument is fed through the quantum memory $\mathcal{N} : A \to B$
- the output of the memory, together with ω_Y^y, are fed into a final measurement $\{\Psi_{BY}^{b|a,\lambda}\}$, and output b is recorded



$$p_q(a,b|x,y) = \sum_\lambda \pi(\lambda)\,\mathrm{Tr}\left[\left(\{(\mathcal{N}_{A\to B}\circ \Phi_{X\to A}^{a|\lambda})(\tau_X^x)\}\otimes \omega_Y^y\right)\Psi_{BY}^{b|a,\lambda}\right]$$

Classical vs Quantum Strategies

Classical:

$$p_c(a,b|x,y) = \sum_\lambda \pi(\lambda)\,\mathrm{Tr}\left[\tau_X^x\ P_X^{a|\lambda}\right]\mathrm{Tr}\left[\omega_Y^y\ Q_Y^{b|a,\lambda})\right]$$

Quantum:

$$p_q(a,b|x,y) = \sum_\lambda \pi(\lambda)\,\mathrm{Tr}\left[\left(\{(\mathcal{N}_{A\to B}\circ\Phi_{X\to A}^{a|\lambda})(\tau_X^x)\}\otimes\omega_Y^y\right)\Psi_{BY}^{b|a,\lambda}\right]$$

Classical vs Quantum

Classical strategies correspond to the case in which the channel $\mathcal{N}$ is entanglement-breaking (i.e., "measure and prepare" form): $\mathcal{N}(\cdot) = \sum_i \rho_i\,\mathrm{Tr}[\cdot\ P_i]$.

Statistical Comparison of Quantum Channels

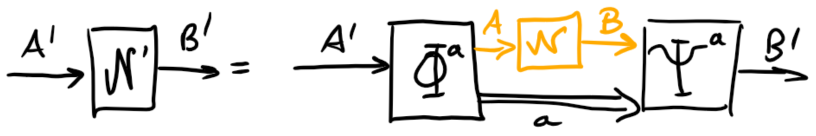
Theorem (Rosset, FB, Liang, 2018)

Given two channels $\mathcal{N} : A \to B$ and $\mathcal{N}' : A' \to B'$, the condition (i.e., "signaling preorder")

$$\mathbb{E}_{\langle \{\tau^x\}, \{\omega^y\}; \mathcal{A}, \mathcal{B}; \ell \rangle}[\mathcal{N}] \geq \mathbb{E}_{\langle \{\tau^x\}, \{\omega^y\}; \mathcal{A}, \mathcal{B}; \ell \rangle}[\mathcal{N}']$$

holds for all semiquantum signaling games, iff there exist a quantum instrument $\{\Phi^a_{A' \to A}\}$ and CPTP maps $\Psi^a_{B \to B'}$ such that

$$\mathcal{N}'_{A' \to B'} = \sum_a \Psi^a_{B \to B'} \circ \mathcal{N}_{A \to B} \circ \Phi^a_{A' \to A} \, .$$



Remarks

- formulation of a resource theory where all and only measure-and-prepare channels are "free"
- any non entanglement-breaking channel can be witnessed
- perfect analogy between separable states and entanglement-breaking channels
- relation with Leggett-Garg inequalities: the "clumsiness loophole" (time-like analogue of communication loophole) can be closed with semiquantum games
- semiquantum games can treat space-like and time-like correlations on an equal footing

Conclusions

Conclusions

- the theory of statistical comparison studies transformations of one "statistical structure" X into another "statistical structure" Y
- equivalent conditions are given in terms of (finitely or infinitely many) *monotones*, e.g., $f_i(X) \geq f_i(Y)$
- such monotones shed light on the "resources" at stake in the operational framework at hand
- in a sense, statistical comparison is complementary to SDP, which instead searches for *efficiently computable* functions like $f(X, Y)$
- however, SDP does not provide much insight into the resources at stake (and not all statistical comparisons are equivalent to SDP!)

Thank you