Quantum Statistical Comparison and Majorization

(and their applications to generalized resource theories)

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Guiding idea:
generalized resource theories as order theories for stochastic (probabilistic) structures

The Precursor: Majorization

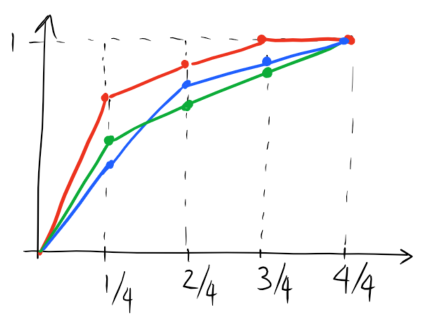
Lorenz Curves and Majorization

- two probability distributions, $\boldsymbol{p} = (p_1, \ldots, p_n)$ and $\boldsymbol{q} = (q_1, \ldots, q_n)$
- truncated sums $P(k) = \sum_{i=1}^{k} p_i^{\downarrow}$ and $Q(k) = \sum_{i=1}^{k} q_i^{\downarrow}$, for all $k = 1, \ldots, n$
- $\boldsymbol{p}$ majorizes $\boldsymbol{q}$, i.e., $\boldsymbol{p} \succeq \boldsymbol{q}$, whenever $P(k) \geq Q(k)$, for all k
- minimal element: uniform distribution $\boldsymbol{e} = n^{-1}(1, 1, \cdots, 1)$

Hardy, Littlewood, and Pólya (1929)

$\boldsymbol{p} \succeq \boldsymbol{q} \iff \boldsymbol{q} = M\boldsymbol{p}$, for some bistochastic matrix M.

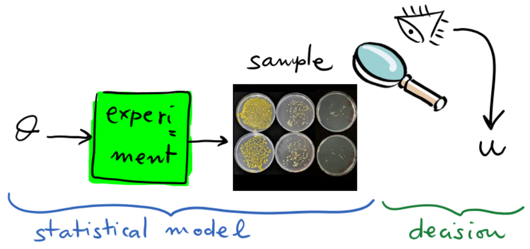
Lorenz curve for probability distribution $\boldsymbol{p} = (p_1, \cdots, p_n)$:



$$(x_k, y_k) = (k/n, P(k)), \quad 1 \leq k \leq n$$

Blackwell's Extensions

Statistical Decision Problems



$$\begin{tikzcd}
\Theta \arrow[r, "\text{experiment}"] \arrow[d, squiggly] & \mathcal{X} \arrow[r, "\text{decision}"] \arrow[d, squiggly] & \mathcal{U} \arrow[d, squiggly] \\
\theta \arrow[r, "w(x|\theta)"'] & x \arrow[r, "d(u|x)"'] & u
\end{tikzcd}$$

payoff is $\ell(\theta, u) \in \mathbb{R}$

Definition

A **statistical model** (or *experiment*) is a triple $\mathbf{w} = \langle \Theta, \mathcal{X}, w \rangle$, a **statistical decision problem** (or *game*) is a triple $\mathbf{g} = \langle \Theta, \mathcal{U}, \ell \rangle$.

Playing Games with Experiments

- the experiment (model) *is given*, i.e., it is the "resource"
- the decision instead *can be optimized*

$$
\begin{tikzcd}
\Theta \arrow[r, "\text{experiment}"] \arrow[d, squiggly] & \mathcal{X} \arrow[r, "\text{decision}"] \arrow[d, squiggly] & \mathcal{U} \arrow[d, squiggly] \\
\theta \arrow[r, "w(x|\theta)"'] & x \arrow[r, "d(u|x)"'] & u
\end{tikzcd}
$$

Definition

The **expected payoff** of a statistical model $\mathbf{w} = \langle \Theta, \mathcal{X}, w \rangle$ w.r.t. a decision problem $\mathbf{g} = \langle \Theta, \mathcal{U}, \ell \rangle$ is given by

$$\mathbb{E}_{\mathbf{g}}[\mathbf{w}] \stackrel{\text{def}}{=} \max_{d(u|x)} \sum_{u,x,\theta} \ell(\theta, u) d(u|x) w(x|\theta) |\Theta|^{-1} .$$

Comparing Statistical Models 1/2

First model: $\mathbf{w} = \langle \Theta, \mathcal{X}, w(x|\theta) \rangle$

$$\begin{tikzcd}
\Theta \arrow[r, "\text{experiment}"] \arrow[d, squiggly] & \mathcal{X} \arrow[r, "\text{decision}"] \arrow[d, squiggly] & \mathcal{U} \arrow[d, squiggly] \\
\theta \arrow[r, "w(x|\theta)"'] & x \arrow[r, "d(u|x)"'] & u
\end{tikzcd}$$

Second model: $\mathbf{w}' = \langle \Theta, \mathcal{Y}, w'(y|\theta) \rangle$

$$\begin{tikzcd}
\Theta \arrow[r, "\text{experiment}"] \arrow[d, squiggly] & \mathcal{Y} \arrow[r, "\text{decision}"] \arrow[d, squiggly] & \mathcal{U} \arrow[d, squiggly] \\
\theta \arrow[r, "w'(y|\theta)"'] & x \arrow[r, "d'(u|y)"'] & u
\end{tikzcd}$$

For a fixed decision problem $\mathbf{g} = \langle \Theta, \mathcal{U}, \ell \rangle$, the expected payoffs $\mathbb{E}_{\mathbf{g}}[\mathbf{w}]$ and $\mathbb{E}_{\mathbf{g}}[\mathbf{w}']$ can always be ordered.

Comparing Statistical Models 2/2

Definition (Information Preorder)

If the model $\mathbf{w} = \langle \Theta, \mathcal{X}, w \rangle$ is better than model $\mathbf{w}' = \langle \Theta, \mathcal{Y}, w' \rangle$ **for all** decision problems $\mathbf{g} = \langle \Theta, \mathcal{U}, \ell \rangle$, then we say that $\mathbf{w}$ is *more informative* than $\mathbf{w}'$, and write

$$\mathbf{w} \succeq \mathbf{w}' \,.$$

Problem. Can we visualize the information morphism $\succeq$ more concretely?

Information Morphism = Statistical Sufficiency

Blackwell-Sherman-Stein Theorem (1948-1953)

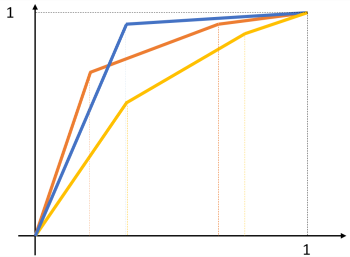
Given two experiments with the same parameter space, $\mathbf{w} = \langle \Theta, \mathcal{X}, w \rangle$ and $\mathbf{w}' = \langle \Theta, \mathcal{Y}, w' \rangle$, the condition $\mathbf{w} \succeq \mathbf{w}'$ holds *iff* there exists a conditional probability $\varphi(y|x)$ such that $w'(y|\theta) = \sum_x \varphi(y|x) w(x|\theta)$.

$$
\begin{array}{ccc}
\Theta & \longrightarrow & \mathcal{Y} \\
\wr\!\downarrow & & \wr\!\downarrow \\
\theta & \underset{w'(y|\theta)}{\longrightarrow} & y
\end{array}
\quad = \quad
\begin{array}{ccccc}
\Theta & \longrightarrow & \mathcal{X} & \xrightarrow{\text{noise}} & \mathcal{Y} \\
\wr\!\downarrow & & \wr\!\downarrow & & \wr\!\downarrow \\
\theta & \underset{w(x|\theta)}{\longrightarrow} & x & \underset{\varphi(y|x)}{\longrightarrow} & y
\end{array}
$$

David H. Blackwell (1919-2010)

Special Case: Dichotomies

- two *pairs* of probability distributions, i.e., two *dichotomies*, $(\boldsymbol{p}_1, \boldsymbol{p}_2)$ and $(\boldsymbol{q}_1, \boldsymbol{q}_2)$, of dimension m and n, respectively
- relabel entries such that ratios p_1^i / p_2^i and q_1^j / q_2^j are nonincreasing
- construct the truncated sums $P_{1,2}(k) = \sum_{i=1}^{k} p_{1,2}^i$ and $Q_{1,2}(k)$
- $(\boldsymbol{p}_1, \boldsymbol{p}_2) \succeq (\boldsymbol{q}_1, \boldsymbol{q}_2)$ iff the relative Lorenz curve of the former is never below that of the latter



Relative Lorenz curves:

$(x_k, y_k) = (P_2(k), P_1(k))$

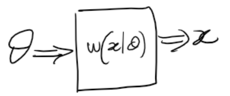
Blackwell's Theorem for Dichotomies (1953)

$(\boldsymbol{p}_1, \boldsymbol{p}_2) \succeq (\boldsymbol{q}_1, \boldsymbol{q}_2) \iff \boldsymbol{q}_i = M\boldsymbol{p}_i$, for some stochastic matrix M.

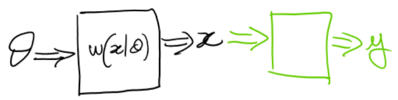
The Viewpoint of Communication Theory

Statistics vs Information Theory

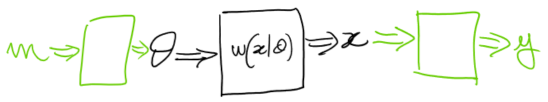
- Statistical models are mathematically equivalent to noisy channels:



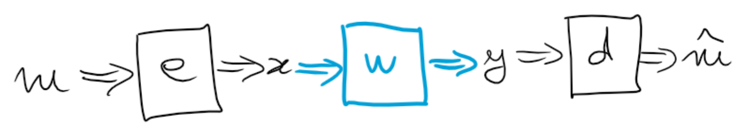
- However: while in statistics the input is inaccessible (Nature does not bother with coding!)



- in communication theory a sender *does code*!



From Decision Problems to Decoding Problems



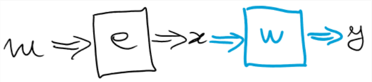
Definition (Decoding Problems)

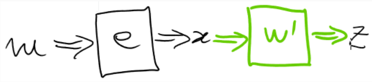
Given a channel $\mathbf{w} = \langle \mathcal{X}, \mathcal{Y}, w(y|x) \rangle$, a decoding problem is defined by an encoding $\mathbf{e} = \langle \mathcal{M}, \mathcal{X}, e(x|m) \rangle$ and the payoff function is the optimum guessing probability:

$$\mathbb{E}_{\mathbf{e}}[\mathbf{w}] \stackrel{\text{def}}{=} \max_{d(m|y)} \sum_{m,x,y} d(m|y) w(y|x) e(x|m) |\mathcal{M}|^{-1} = 2^{-H_{\min}(M|Y)}$$

Comparison of Classical Noisy Channels

Consider two discrete noisy channels **w** and **w**′ with the same input alphabet





Theorem

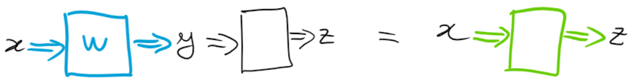
Given the following pre-orders

1. degradability: there exists $\varphi(z|y)$: $w'(x|z) = \sum_y \varphi(z|y) w(y|x)$
2. noisiness: for all encodings **e** $= \langle \mathcal{M}, \mathcal{X}, e(x|m) \rangle$, $H(M|Y) \leq H(M|Z)$
3. ambiguity: for all encodings **e** $= \langle \mathcal{M}, \mathcal{X}, e(x|m) \rangle$, $H_{\min}(M|Y) \leq H_{\min}(M|Z)$

we have: (1) $\Longrightarrow$ (2) (data-processing inequality), (2) $\not\Longrightarrow$ (1) (Körner and Marton, 1977), but (1) $\Longleftrightarrow$ (3) (FB, 2016).

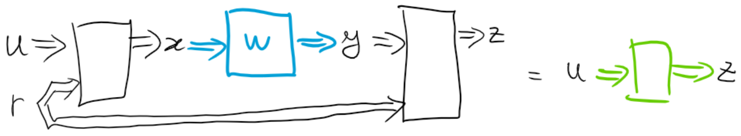
Some Classical Channel Morphisms

Output degrading:



Input degrading:

Full coding (Shannon's "channel inclusion", 1958):

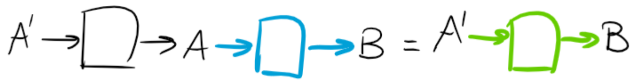


Extensions to the Quantum Case

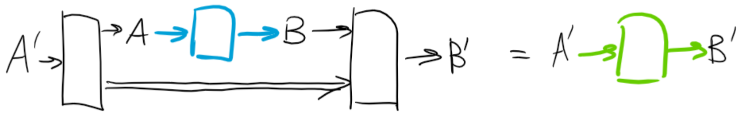
Some Quantum Channel Morphisms

Output degrading:

Input degrading:



Quantum coding with forward CC:



Output Degradability

Comparison of Quantum Statistical Models 1/2

Quantum statistical models as cq-channels:

Formulation below from: A.S. Holevo, *Statistical Decision Theory for Quantum Systems*, 1973.

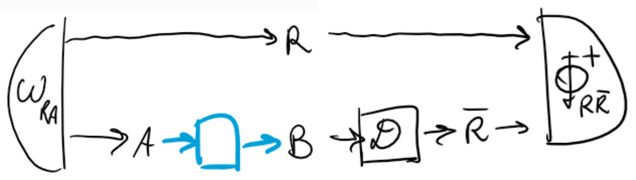
classical case	**quantum case**
• decision problems $\mathbf{g} = \langle \Theta, \mathcal{U}, \ell \rangle$	• decision problems $\mathbf{g} = \langle \Theta, \mathcal{U}, \ell \rangle$
• experiments $\mathbf{w} = \langle \Theta, \mathcal{X}, \{w(x\|\theta)\} \rangle$	• quantum experiments $\mathcal{E} = \left\langle \Theta, \mathcal{H}_S, \{\rho_S^\theta\} \right\rangle$
• decisions $d(u\|x)$	• POVMs $\{P_S^u : u \in \mathcal{U}\}$
• $p_c(u,\theta) = \sum_x d(u\|x) w(x\|\theta) \|\Theta\|^{-1}$	• $p_q(u,\theta) = \mathrm{Tr}\left[\rho_S^\theta \; P_S^u\right] \|\Theta\|^{-1}$
• $\mathbb{E}_{\mathbf{g}}[\mathbf{w}] = \max_{d(u\|x)} \sum \ell(\theta,u) p_c(u,\theta)$	• $\mathbb{E}_{\mathbf{g}}[\mathcal{E}] = \max_{\{P_S^u\}} \sum \ell(\theta,u) p_q(u,\theta)$

Comparison of Quantum Statistical Models 2/2

What follows is from: FB, Comm. Math. Phys., 2012

- consider two quantum statistical models $\mathcal{E} = \langle \Theta, \mathcal{H}_S, \{\rho_S^\theta\} \rangle$ and $\mathcal{E}' = \langle \Theta, \mathcal{H}_{S'}, \{\sigma_{S'}^\theta\} \rangle$
- **information ordering**: $\mathcal{E} \succeq \mathcal{E}'$ iff $\mathbb{E}_{\mathbf{g}}[\mathcal{E}] \geq \mathbb{E}_{\mathbf{g}}[\mathcal{E}']$ for all $\mathbf{g}$
- $\mathcal{E} \succeq \mathcal{E}'$ iff there exists a **quantum statistical morphism** (essentially, a PTP map) $\mathcal{M} : \mathsf{L}(\mathcal{H}_S) \to \mathsf{L}(\mathcal{H}_{S'})$ such that $\mathcal{M}(\rho_S^\theta) = \sigma_{S'}^\theta$ for all θ
- **complete information ordering**: $\mathcal{E} \succeq_c \mathcal{E}'$ iff $\mathcal{E} \otimes \mathcal{F} \succeq \mathcal{E}' \otimes \mathcal{F}$ for all ancillary models $\mathcal{F}$ (in fact, one informationally complete model suffices)
- $\mathcal{E} \succeq_c \mathcal{E}'$ iff there exists a **CPTP map** $\mathcal{N} : \mathsf{L}(\mathcal{H}_S) \to \mathsf{L}(\mathcal{H}_{S'})$ such that $\mathcal{N}(\rho_S^\theta) = \sigma_{S'}^\theta$ for all θ
- if $\mathcal{E}'$ is **abelian**, then $\mathcal{E} \succeq_c \mathcal{E}'$ iff $\mathcal{E} \succeq \mathcal{E}'$

Comparison of Quantum Channels 1/2



Definition (Quantum Decoding Problems)

Given a quantum channel $\mathcal{N} : A \to B$, a quantum decoding problem is defined by a bipartite state ω_{RA} and the payoff function is the optimum singlet fraction:

$$\mathbb{E}_\omega[\mathcal{N}] \stackrel{\text{def}}{=} \max_{\mathcal{D}} \langle \Phi^+_{R\bar{R}} | (\mathrm{id}_R \otimes \mathcal{D}_{B\to\bar{R}} \circ \mathcal{N}_{A\to B})(\omega_{RA}) | \Phi^+_{R\bar{R}} \rangle$$

Comparison of Quantum Channels 2/2

Theorem (FB, 2016)

Given two quantum channels $\mathcal{N} : A \to B$ and $\mathcal{N}' : A \to B'$, the following are equivalent:

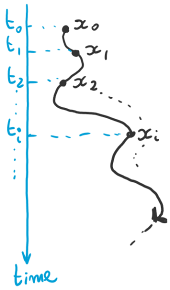
1. **output degradability**: *there exists CPTP map $\mathcal{C}$: $\mathcal{N}' = \mathcal{C} \circ \mathcal{N}$;*
2. **coherence preorder**: *for any bipartite state ω_{RA}, $\mathbb{E}_\omega[\mathcal{N}] \geq \mathbb{E}_\omega[\mathcal{N}']$, that is,* $H_{\min}(R|B)_{(\mathsf{id}\otimes\mathcal{N})(\omega)} \leq H_{\min}(R|B')_{(\mathsf{id}\otimes\mathcal{N}')(\omega)}$.

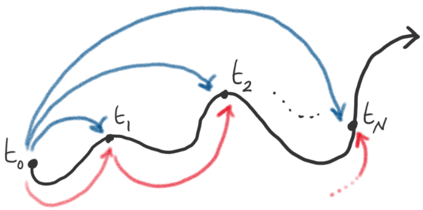
$\rightsquigarrow$ applications to the theory of open quantum systems dynamics and, by adding symmetry constraints, to quantum thermodynamics

Application 1:
Open Quantum Systems Dynamics

Discrete-Time Stochastic Processes

- Let x_i, for $i = 0, 1, \ldots,$ index the state of a system at time $t = t_i$
- if the system can be initialized at time $t = t_0$, the process is fully described by the conditional distribution $p(x_N, \ldots, x_1|x_0)$
- if the system evolving is quantum, we only have a quantum dynamical mapping $\left\{\mathcal{N}^{(i)}_{Q_0 \to Q_i}\right\}_{i=1,\ldots,N}$
- the process is divisible if there exist channels $\mathcal{D}^{(i)}$ such that $\mathcal{N}^{(i+1)} = \mathcal{D}^{(i)} \circ \mathcal{N}^{(i)}$ for all i
- problem: to provide a fully information-theoretic characterization of divisibility



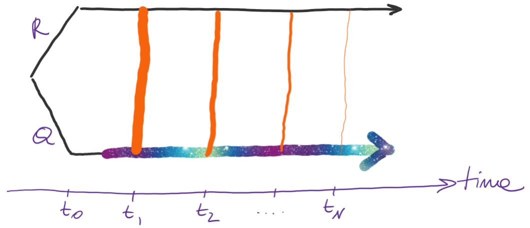


Divisibility as "Quantum Information Flow"

Theorem (2016-2018)

Given an initial open quantum system Q_0, *a quantum dynamical mapping* $\left\{\mathcal{N}^{(i)}_{Q_0\to Q_i}\right\}_{i\geq 1}$ *is divisibile if and only if, for any initial state* ω_{RQ_0},

$$H_{\min}(R|Q_1) \leq H_{\min}(R|Q_2) \leq \cdots \leq H_{\min}(R|Q_N) .$$



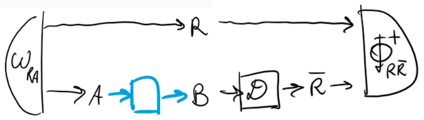
Application 2:
Quantum Thermodynamics

Resource Theory of Athermality and Asymmetry

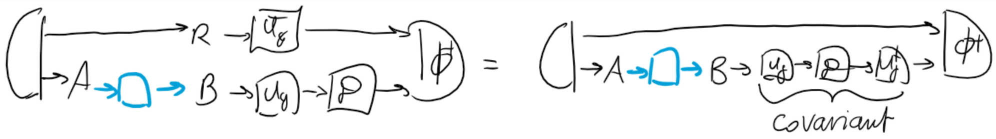
From [FB, arXiv:1505.00535], [FB and Gour, Phys. Rev. A 95, 012110 (2017)], and [Gour, Jennings, FB, Duan, and Marvian, Nat. Comm. 9, 5352 (2018)]

- idea: to characterize thermal accessibility $\rho \to \sigma$ by comparing the dichotomies (ρ, γ) and (σ, γ), for γ thermal state
- classically, Blackwell's theorem implies the thermomajorization relation
- in the quantum case, in order to account for coherence, symmetry constraints can also be added to the Gibbs-preserving map

Sketch Idea



- we compare the singlet fraction of two channels, $\mathcal{N}^i_{A\to B}(\bullet) = \langle 0| \bullet |0\rangle \gamma + \langle 1| \bullet |1\rangle \rho^i$, with $\rho^1 \equiv \rho$ and $\rho^2 \equiv \sigma$
- to add symmetry constraints, we compare the two channels for the *twirled* quantum codes:

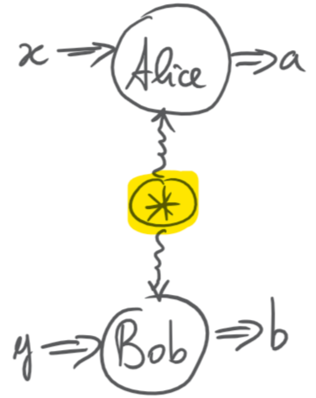


- by varying the input quantum code, we obtain a complete set of entropic monotones

Quantum Coding:
Probing Quantum Correlations in Space-Time

Part One: Quantum Space-Like Correlations

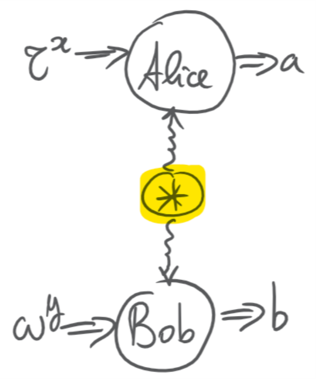
- nonlocal games (Bell tests) can be seen here as bipartite decision problems **ng** $= \langle \mathcal{X}, \mathcal{Y}; \mathcal{A}, \mathcal{B}; \ell \rangle$ played "in parallel" by non-communicating players
- with a classical source, $p_c(a, b|x, y) = \sum_\lambda \pi(\lambda) d_A(a|x, \lambda) d_B(b|y, \lambda)$
- with a quantum source, $p_q(a, b|x, y) = \mathrm{Tr}\left[\rho_{AB} \left(P_A^{a|x} \otimes Q_B^{b|y}\right)\right]$



$$\mathbb{E}_{\mathsf{nl}}[*] \stackrel{\mathsf{def}}{=} \max \sum_{x,y,a,b} \ell(x, y; a, b) p_{c/q}(a, b|x, y) |\mathcal{X}|^{-1} |\mathcal{Y}|^{-1}$$

Semiquantum Nonlocal Games

- **semiquantum nonlocal games** replace classical inputs with quantum inputs: **sqnl** $= \langle \{\tau^x\}, \{\omega^y\}; \mathcal{A}, \mathcal{B}; \ell \rangle$
- with a classical source,
  $p_c(a, b|x, y) = \sum_\lambda \pi(\lambda) \operatorname{Tr}\left[(\tau^x_X \otimes \omega^y_Y)\ (P^{a|\lambda}_X \otimes Q^{b|\lambda}_Y)\right]$
- with a quantum source,
  $p_q(a, b|x, y) = \operatorname{Tr}\left[(\tau^x_X \otimes \rho_{AB} \otimes \omega^y_Y)\ (P^a_{XA} \otimes Q^b_{BY})\right]$



$$\mathbb{E}_{\textbf{sqnl}}[*] \stackrel{\text{def}}{=} \max \sum_{x,y,a,b} \ell(x, y; a, b) p_{c/q}(a, b|x, y) |\mathcal{X}|^{-1} |\mathcal{Y}|^{-1}$$

LOSR Morphisms of Quantum Correlations

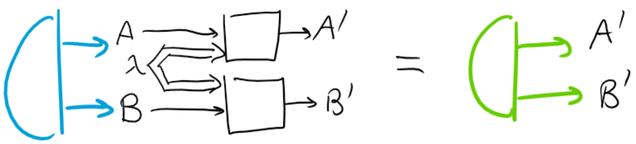
Theorem (FB, 2012)

Given two bipartite states ρ_{AB} and $\sigma_{A'B'}$, the condition (i.e., "nonlocality preorder")

$$\mathbb{E}_{\textbf{sqnl}}[\rho_{AB}] \geq \mathbb{E}_{\textbf{sqnl}}[\sigma_{A'B'}]$$

holds for all semiquantum nonlocal games **sqnl** $= \langle\{\tau^x\},\{\omega^y\};\mathcal{A},\mathcal{B};\ell\rangle$,
iff there exist CPTP maps $\{\Phi^\lambda_{A\to A'}\}$, $\{\Psi^\lambda_{B\to B'}\}$, *and distribution* $\pi(\lambda)$ *such that*

$$\sigma_{A'B'} = \sum_\lambda \pi(\lambda)(\Phi^\lambda_{A\to A'} \otimes \Psi^\lambda_{B\to B'})(\rho_{AB}) \ .$$



Corollaries

- For any separable state ρ_{AB},

$$\mathbb{E}_{\textbf{sqnl}}[\rho_{AB}] = \mathbb{E}_{\textbf{sqnl}}[\rho_A \otimes \rho_B] = \mathbb{E}_{\textbf{sqnl}}^{\text{sep}}\ ,$$

  for all semiquantum nonlocal games $\textbf{sqnl} = \langle \{\tau^x\}, \{\omega^y\}; \mathcal{A}, \mathcal{B}; \ell \rangle$.
- For any entangled state ρ_{AB}, **there exists** a semiquantum nonlocal game $\textbf{sqnl} = \langle \{\tau^x\}, \{\omega^y\}; \mathcal{A}, \mathcal{B}; \ell \rangle$ such that

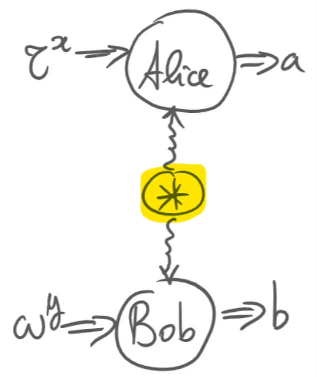
$$\mathbb{E}_{\textbf{sqnl}}[\rho_{AB}] > \mathbb{E}_{\textbf{sqnl}}^{\text{sep}}\ .$$

Other Properties of Semiquantum Nonlocal Games

From [Branciard, Rosset, Liang, and Gisin, Phys. Rev. Lett. 110, 060405 (2013)]

Semiquantum nonlocal games:

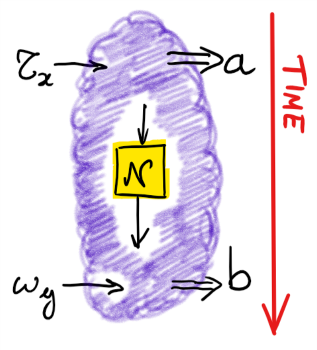
- can be considered as measurement device-independent entanglement witnesses (i.e., MDI-EW)
- can withstand losses in the detectors
- can withstand any amount of classical communication exchanged between Alice and Bob
- hence, contrarily to conventional Bell tests, semiquantum nonlocal games are non trivial also when rearranged *in time*



Part Two: Quantum Time-Like Correlations

Semiquantum *signaling* games:

- the duo Alice–Bob becomes 'Alice now'–'Alice later'
- the semiquantum nonlocal game **sqnl** $= \langle \{\tau^x\}, \{\omega^y\}; \mathcal{A}, \mathcal{B}; \ell \rangle$ is arranged in a time-like structure
- thus obtaining a semiquantum signaling game **sqsg**
- with unlimited classical memory, $p_c(a, b|x, y) = \sum_\lambda \pi(\lambda) \operatorname{Tr}\left[\tau_X^x \; P_X^{a|\lambda}\right] \operatorname{Tr}\left[\omega_Y^y \; Q_Y^{b|a,\lambda}\right]$
- if, moreover, a quantum memory $\mathcal{N} : A \to B$ is available?

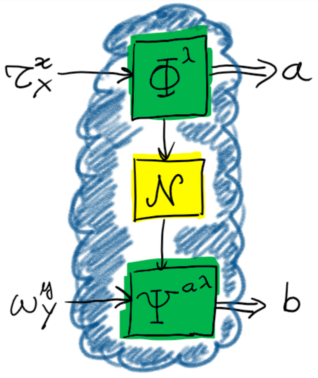


Admissible Quantum Strategies

- τ_X^x is fed through an *instrument* $\{\Phi_{X\to A}^{a|\lambda}\}$, and outcome a is recorded

- the quantum output of the instrument is fed through the quantum memory $\mathcal{N} : A \to B$

- the output of the memory, together with ω_Y^y, are fed into a final measurement $\{\Psi_{BY}^{b|a,\lambda}\}$, and output b is recorded



$$p_q(a,b|x,y) = \sum_\lambda \pi(\lambda)\,\mathrm{Tr}\Big[\Big(\{(\mathcal{N}_{A\to B} \circ \Phi_{X\to A}^{a|\lambda})(\tau_X^x)\} \otimes \omega_Y^y\Big)\,\Psi_{BY}^{b|a,\lambda}\Big]$$

Classical vs Quantum Strategies

Classical:

$$p_c(a,b|x,y) = \sum_\lambda \pi(\lambda)\,\mathrm{Tr}\left[\tau_X^x \; P_X^{a|\lambda}\right]\mathrm{Tr}\left[\omega_Y^y \; Q_Y^{b|a,\lambda}\right]$$

Quantum:

$$p_q(a,b|x,y) = \sum_\lambda \pi(\lambda)\,\mathrm{Tr}\left[\left(\{(\mathcal{N}_{A\to B}\circ\Phi_{X\to A}^{a|\lambda})(\tau_X^x)\}\otimes\omega_Y^y\right)\Psi_{BY}^{b|a,\lambda}\right]$$

Classical vs Quantum

Classical strategies correspond to the case in which the channel $\mathcal{N}$ is entanglement-breaking (i.e., "measure and prepare" form): $\mathcal{N}(\cdot) = \sum_i \rho_i \,\mathrm{Tr}[\cdot\; P_i]$.

EB Morphisms of Quantum Channels

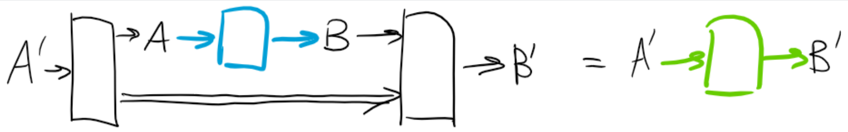
Theorem (Rosset, FB, Liang, 2018)

Given two channels $\mathcal{N} : A \to B$ *and* $\mathcal{N}' : A' \to B'$*, the condition (i.e., "quantum signaling preorder")*

$$\mathbb{E}_{\textbf{sqsg}}[\mathcal{N}] \geq \mathbb{E}_{\textbf{sqsg}}[\mathcal{N}']$$

holds for all semiquantum signaling games **sqsg** $= \langle \{\tau^x\}, \{\omega^y\}; \mathcal{A}, \mathcal{B}; \ell \rangle$*,*
iff there exist a quantum instrument $\{\Phi^a_{A' \to A}\}$ *and CPTP maps* $\{\Psi^a_{B \to B'}\}$ *such that*

$$\mathcal{N}'_{A' \to B'} = \sum_a \Psi^a_{B \to B'} \circ \mathcal{N}_{A \to B} \circ \Phi^a_{A' \to A} \, .$$



A Resource Theory of Quantum Memories: Some Remarks

- formulation of a resource theory where all and only measure-and-prepare channels are “free”
- any non entanglement-breaking channel can be witnessed
- perfect analogy between separable states and entanglement-breaking channels
- relation with Leggett-Garg inequalities: the “clumsiness loophole” (time-like analogue of communication loophole) can be closed with semiquantum games
- semiquantum games can treat space-like and time-like correlations on an equal footing

Conclusions

Conclusions

- the theory of statistical comparison studies morphisms (preorders) of one "statistical structure" X into another "statistical structure" Y
- equivalent conditions are given in terms of (finitely or infinitely many) *monotones*, e.g., $f_i(X) \geq f_i(Y)$
- such monotones shed light on the "resources" at stake in the operational framework at hand
- in a sense, statistical comparison is complementary to SDP, which instead searches for *efficiently computable* functions like $f(X, Y)$
- however, SDP does not provide much insight into the resources at stake (and not all statistical comparisons are equivalent to SDP!)

Thank you